Name: _____

TOPIC TEST

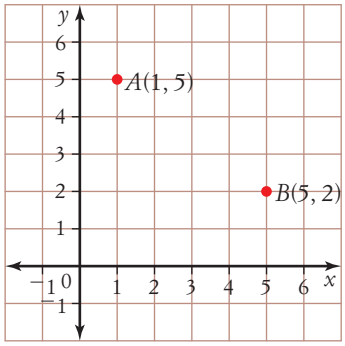
Vectors

- Time allowed: 45 minutes
- Part A: 10 multiple-choice questions (10 marks)
- Part B: 7 free-response questions (40 marks)
- Total: 50 marks

Part A

10 multiple-choice questions
1 mark each: 10 marks
Circle the correct answer.

1 Given the points A and B shown, then $\mathbf{AB} =$



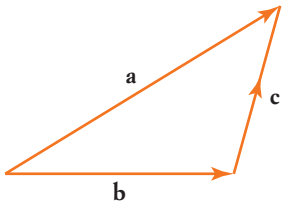
A $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$

B $\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

C $\begin{bmatrix} 3 \\ -4 \end{bmatrix}$

D $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$

2 Given the three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ shown, $\mathbf{c} =$



A $\mathbf{a} + \mathbf{b}$

B $\mathbf{a} - \mathbf{b}$

C $\mathbf{b} - \mathbf{a}$

D $\mathbf{b} + \mathbf{a}$

3 Given $\mathbf{p} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, what is the opposite of $\mathbf{p}$?

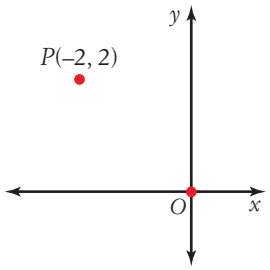
A $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

B $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

C $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

D $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

4 Given the point P and the origin O shown, then **OP** in polar form is



A $\begin{bmatrix} 2\sqrt{2} \\ \frac{3\pi}{4} \end{bmatrix}$

B $\begin{bmatrix} 2\sqrt{2} \\ \frac{7\pi}{4} \end{bmatrix}$

C $\begin{bmatrix} 2\sqrt{2} \\ -\frac{\pi}{4} \end{bmatrix}$

D $\begin{bmatrix} 2\sqrt{2} \\ \frac{\pi}{4} \end{bmatrix}$

5 $\begin{bmatrix} 1 \\ \frac{11\pi}{6} \end{bmatrix}$ is equivalent to the vector

A $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$

B $\begin{bmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$

C $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}$

D $\begin{bmatrix} -\frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}$

6 The unit vector $\hat{\mathbf{a}}$ when $\mathbf{a} = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$ is

A $\begin{bmatrix} 0.8 \\ -0.6 \end{bmatrix}$

B $\begin{bmatrix} -0.8 \\ -0.6 \end{bmatrix}$

C $\begin{bmatrix} 0.6 \\ -0.8 \end{bmatrix}$

D $\begin{bmatrix} -0.6 \\ -0.8 \end{bmatrix}$

7 $\begin{bmatrix} 10 \\ \frac{\pi}{3} \end{bmatrix} + \begin{bmatrix} 10 \\ -\frac{\pi}{3} \end{bmatrix} =$

A $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

B $\begin{bmatrix} 10 \\ \sqrt{3} \end{bmatrix}$

C $\begin{bmatrix} 10 \\ 0 \end{bmatrix}$

D $\begin{bmatrix} 20 \\ 0 \end{bmatrix}$

8 Express the answer to $\begin{bmatrix} 8 \\ 270^\circ \end{bmatrix} + \begin{bmatrix} 4 \\ 180^\circ \end{bmatrix}$ in **i**-**j** form.

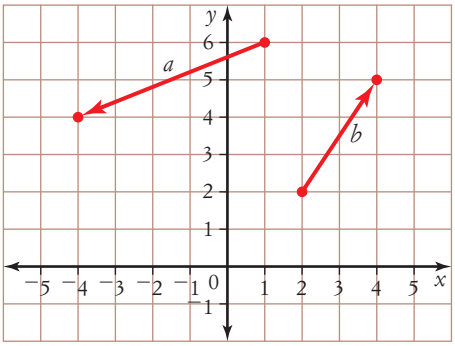
A $-4\mathbf{i} - 8\mathbf{j}$

B $8\mathbf{i} + 4\mathbf{j}$

C $8\mathbf{i} - 4\mathbf{j}$

D $-4\mathbf{i} + 8\mathbf{j}$

9 Express $\mathbf{a} + \mathbf{b}$ in **i**-**j** form given the vectors below.



A $3\mathbf{i} + \mathbf{j}$

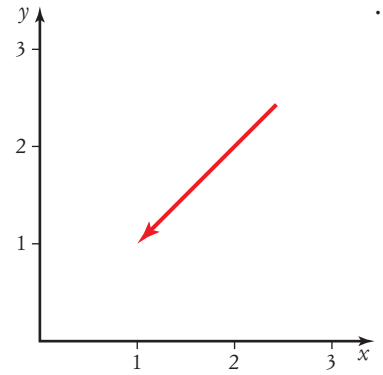
B $3\mathbf{i} - \mathbf{j}$

C $-3\mathbf{i} + \mathbf{j}$

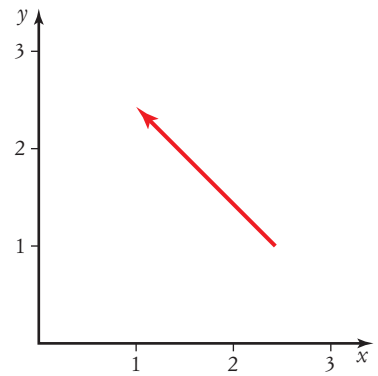
D $-3\mathbf{i} - \mathbf{j}$

10 The sketch of a force equal to 2 N acting in a north-easterly direction is

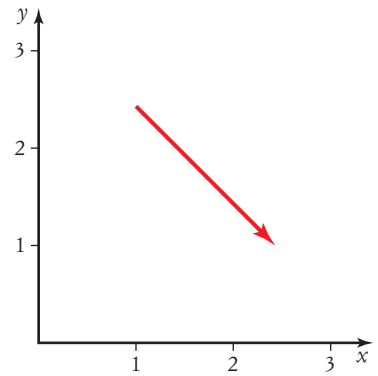
A



B



C



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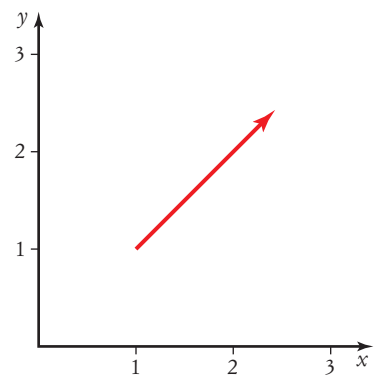


Part B

7 free-response questions
40 marks
Show your working where appropriate.

11 Given $\mathbf{a} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$, then simplify each expression.

a $3\mathbf{a} - 2\mathbf{b} + \mathbf{c}$

b $10\mathbf{a} - 5\mathbf{c}$

[4 marks]

12 Given $\mathbf{a} = \begin{bmatrix} 5 \\ -12 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -12 \\ 5 \end{bmatrix}$, find:

a $|\mathbf{a}|$

b $\hat{\mathbf{a}} + \hat{\mathbf{b}}$

c r and θ if $\begin{bmatrix} 5 \\ -12 \end{bmatrix} = \begin{bmatrix} r \\ \theta \end{bmatrix}$.

[6 marks]

13 A boy runs 3 kilometres due north, 5 kilometres south-east then 4 kilometres west.

a Determine the displacement vector.

b Hence determine the distance from the starting point.

[5 marks]

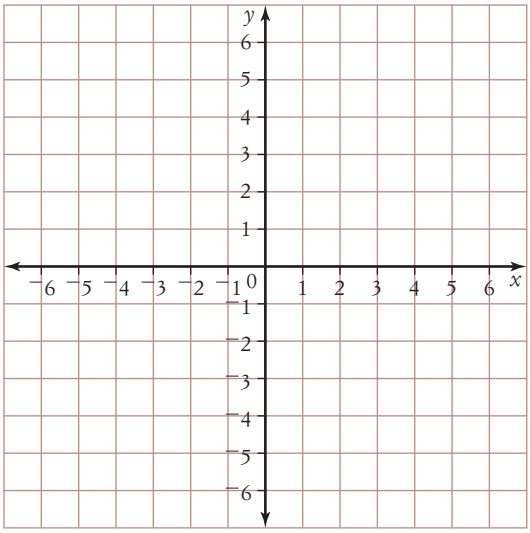
14 a Express the vectors $\mathbf{p} = \begin{bmatrix} 8 \\ \frac{\pi}{6} \end{bmatrix}$ and $\mathbf{q} = \begin{bmatrix} 8 \\ \frac{2\pi}{3} \end{bmatrix}$ in component form and hence find $\mathbf{p} + \mathbf{q}$.

b Hence determine $|\mathbf{p} + \mathbf{q}|$.

[6 marks]

15 Consider the vectors $\mathbf{a} = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

a Sketch $-\frac{\mathbf{a}}{2}$ and $3\mathbf{b}$ on the set of axes below.

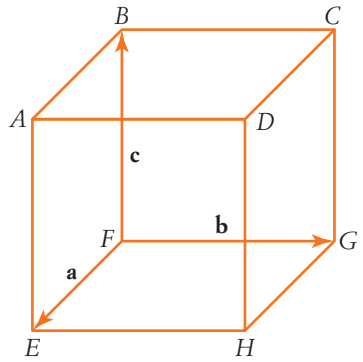


b Show that $|\mathbf{a}| = 3|\mathbf{b}|$.

c Convert $-2\mathbf{a} + 3\mathbf{b}$ to polar form.

[10 marks]

16 Given this cube with **FE** = **a**, **FB** = **b** and **FB** = **c**, express **AG** in terms of **a**, **b** and **c**.



[3 marks]

17 The air speed of an aeroplane is 230 km/h. A steady wind is blowing from the south-west at 30 km/h.

a In what direction should the plane head to fly to *B*, which is due east of *A*?

b How long will the airplane take to travel to *B* if *B* is 350 km from *A*?

[6 marks]

This is the end of the test.
Use the rest of this page for extra working space.

Answers

1 C **2** B **3** C **4** A **5** C
6 B **7** C **8** A **9** C **10** D

11 a $3\mathbf{a}-2\mathbf{b}+\mathbf{c}=3\begin{bmatrix}1\\-3\end{bmatrix}-2\begin{bmatrix}2\\3\end{bmatrix}+\begin{bmatrix}5\\-2\end{bmatrix}=\begin{bmatrix}3\\-9\end{bmatrix}-\begin{bmatrix}4\\6\end{bmatrix}+\begin{bmatrix}5\\-2\end{bmatrix}=\begin{bmatrix}4\\-17\end{bmatrix}$

b $10\mathbf{a}-5\mathbf{c}=10\begin{bmatrix}1\\-3\end{bmatrix}-5\begin{bmatrix}5\\-2\end{bmatrix}$

$=\begin{bmatrix}10\\-30\end{bmatrix}-\begin{bmatrix}25\\-10\end{bmatrix}$

$=\begin{bmatrix}-15\\-20\end{bmatrix}$

12 a $|\mathbf{a}|=\left\|\begin{bmatrix}5\\-12\end{bmatrix}\right\|=\sqrt{25+144}=\sqrt{169}=13$

b $\hat{\mathbf{a}}+\hat{\mathbf{b}}=\frac{1}{13}(5\mathbf{i}-12\mathbf{j})+\frac{1}{13}(-12\mathbf{i}+5\mathbf{j})$

$=\frac{1}{13}(-7\mathbf{i}-7\mathbf{j})$

$\hat{\mathbf{a}}+\hat{\mathbf{b}}=-\frac{7}{13}(\mathbf{i}+\mathbf{j})$

c $\left\|\begin{bmatrix}5\\-12\end{bmatrix}\right\|=13 \ \therefore r=13$

$\tan(\theta)=-\frac{12}{5}$

$\theta=-1.176$

$\therefore \mathbf{p}+\mathbf{q}=\begin{bmatrix}13\\-1.176\end{bmatrix}$

13 a $\begin{bmatrix}0\\3\end{bmatrix}+\begin{bmatrix}5\cos(-45°)\\5\sin(-45°)\end{bmatrix}+\begin{bmatrix}-4\\0\end{bmatrix}=\begin{bmatrix}-0.464\\-0.536\end{bmatrix}$

b $\left\|\begin{bmatrix}-0.464\\-0.536\end{bmatrix}\right\|=0.709$

14 a $\mathbf{p} = \begin{bmatrix} 8 \\ \frac{\pi}{6} \end{bmatrix} = \begin{bmatrix} 8\cos\left(\frac{\pi}{6}\right) \\ 8\sin\left(\frac{\pi}{6}\right) \end{bmatrix} = \begin{bmatrix} 4\sqrt{3} \\ 4 \end{bmatrix}$

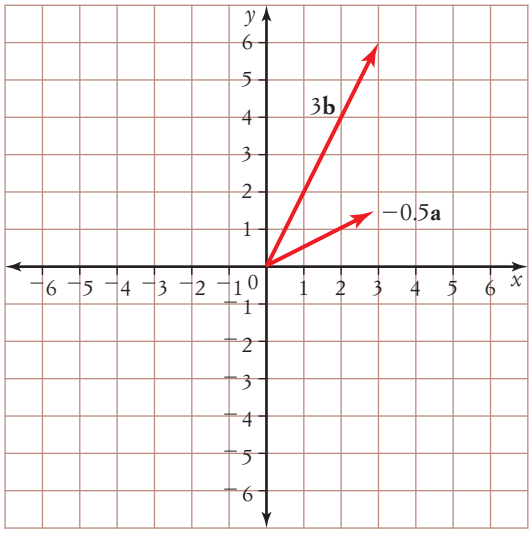
$\mathbf{q} = \begin{bmatrix} 8 \\ \frac{2\pi}{3} \end{bmatrix} = \begin{bmatrix} 8\cos\left(\frac{2\pi}{3}\right) \\ 8\sin\left(\frac{2\pi}{3}\right) \end{bmatrix} = \begin{bmatrix} -4 \\ 4\sqrt{3} \end{bmatrix}$

$\mathbf{p} + \mathbf{q} = \begin{bmatrix} 4\sqrt{3} \\ 4 \end{bmatrix} + \begin{bmatrix} -4 \\ 4\sqrt{3} \end{bmatrix} = \begin{bmatrix} 4(\sqrt{3}-1) \\ 4(1+\sqrt{3}) \end{bmatrix}$

b $|\mathbf{p} + \mathbf{q}| = 11.314$

15 a $-\frac{\mathbf{a}}{2} = -\frac{1}{2}\begin{bmatrix} -6 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1.5 \end{bmatrix}$ and

$3\mathbf{b} = 3\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$



b Show that $|\mathbf{a}| = 3|\mathbf{b}|$.

$|\mathbf{a}| = \sqrt{36+9} = \sqrt{45}$

$3|\mathbf{b}| = 3\sqrt{1+4} = \sqrt{9}\sqrt{5} = \sqrt{45}$

$\therefore |\mathbf{a}| = 3|\mathbf{b}|$

c $-2\mathbf{a} + 3\mathbf{b} = -2\begin{bmatrix} -6 \\ -3 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 15 \\ 12 \end{bmatrix}$

$r = \left|\begin{bmatrix} 15 \\ 12 \end{bmatrix}\right| = \sqrt{369} = 19.209$

$\theta = \tan^{-1}\left(\frac{12}{15}\right) = 0.675$

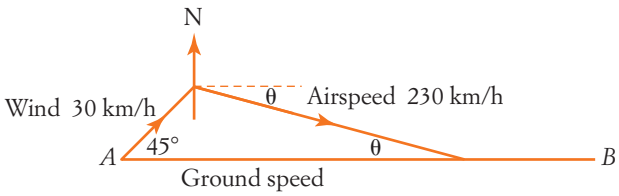
$\therefore -2\mathbf{a} + 3\mathbf{b} \equiv \begin{bmatrix} \sqrt{369} \\ 0.675 \end{bmatrix}$

16 $\mathbf{AG} = \mathbf{AF} + \mathbf{FG}$

$\mathbf{AG} = \mathbf{AE} + \mathbf{EF} + \mathbf{FG}$

$\mathbf{AG} = -\mathbf{c} - \mathbf{a} + \mathbf{b}$

17 a



$\frac{\sin(\theta)}{30} = \frac{\sin(45°)}{230}$

$\sin(\theta) = 0.092\,231\,392\,9$

$\theta = 5.29°$

Direction the plane must go is $90° + 5.29° = 95.29° = 095°$.

b Let d be the ground speed.

$\frac{d}{\sin(45° + 90° - 5.29°)} = \frac{230}{\sin(45°)}$

$d = 250.233$

$\text{Time} = \frac{350}{250.233} = 1.399$ hours

The plane will take 1 hour 24 minutes.